An examination of the ratio in Eq. (17) indicates that if either $F \to \infty$ or $G \to 0$, then $\dot{a} \to 0$. The first alternative is not possible, but the second is. Thus, the condition for $\dot{a}=0$ can be taken to be $G(a)=0$. Examination of Eq. (16) indicates that this condition leads to the requirement that

$$F=\frac{\pi^2 kh^2}{16L}a(a+2a_0) \tag{18}$$

Thus, the combinations of values of F, a, k, and a_0 for which $\dot{a} \to 0$ can be determined.

Discussion

Nonlinear constitutive laws produce softening effects which lead to finite collapse times in column creep. If nonlinear geometrical effects are included in the analysis, collapse times are either increased or eliminated. Two geometrical effects which can produce stiffening are the nonlinear strain-curvature relation studied by Huang[4] and the end restraint effect examined in this Note.

Finally, it should be noted that although the stabilizing influence of nonlinear geometric effects might alleviate a concern for collapse, Böstrom[5] indicates that another failure mode should also be considered. He showed that when creep-induced embrittlement processes are operative, fracture may be the mode of failure. Creep buckling experiments on polypropylene plastic columns at room temperature have, in fact, been observed to terminate ultimately by fracture.[6] Because many heat resistant alloys are subject to embrittlement, the fracture mode of failure should be considered a distinct possibility.

References

[1] Ross, A. D., "The Effects of Creep on Instability and Indeterminacy Investigated by Plastic Models," *The Structural Engineer*, Vol. 24, No. 8, Aug. 1946, p. 413.

[2] Hoff, N. J., "A Survey of the Theories of Creep Buckling," *Proceedings of the Third U. S. National Congress of Applied Mechanics*, ASME, 1958, pp. 29-49.

[3] Hult, J., *Creep in Engineering Structures*, Blaisdell Publishing Co., Waltham, Mass., 1966, pp. 61-71.

[4] Huang, N. C., "Creep Buckling of Imperfect Columns," *Journal of Applied Mechanics*, March 1976, pp. 131-136.

[5] Böstrom, P. O., "Creep Buckling Considering Material Damage," *International Journal of Solids and Structures*, Vol. 11, June 1975, pp. 765-775.

[6] Morgan, A. G. Jr., "Column Creep Buckling With Elastic End Restraint," Special Problem Submitted to School of Aerospace Engineering, Georgia Institute of Technology, April 1980.

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Effective Radiation Scattering Coefficient

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Nomenclature

c' = effective scattering coefficient, $=C_{sca}/V_p$
C_{sca} = scattering cross section
D = scattering particle diameter; particle is a sphere
I = intensity
ℓ = path length
N_0 = particle number density
n = relative index of refraction
T = transmittance
V_p = scattering particle volume, $=\pi D^3/6$
x = size parameter, $n_w \pi D/\lambda_0$; n_w is water refractive index
η = particle volume concentration
λ_0 = wavelength in vacuum, 0.6328 μm
τ = optical thickness, $=N_0 C_{sca}\ell$

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Introduction

In previously reported work[1-3] an effective scattering coefficient c' was presented. This coefficient, although convenient from an experimental point of view, may be a bit unusual since it is the ratio of scattering cross section C_{sca} and the scattering particle volume V_p. The purpose of this Note is to present the variation of this coefficient as a function of particle size assuming a purely scattering media. Once obtained, this coefficient could then be used to determine the scattering cross section. Instead, in this presentation, the values of the theoretical and experimental effective scattering coefficient as a function of scattering particle diameter are compared in both tabular and graphical forms.

Discussion

Transmission data have been obtained for an assumed purely scattering media using 11 sets of uniformally sized polystyrene latex spheres as scattering centers. The latex spheres were diluted with distilled water to produce solutions of known concentrations. Transmission measurements of these concentrations at one wavelength (0.6328 μm) were then obtained. The spectrophotometer used had a path length of 1 cm and measured the transmission relative to another cell containing distilled water only.

The transmission of a scattering media is a function of the scattering sphere number density N_0, the scattering cross section C_{sca}, and path length ℓ

$$T=\exp(-\tau)=\exp(-N_0 C_{sca}\ell) \tag{1}$$

The scattering medium was characterized by the scattering spheres volume concentration η, which is the ratio of the volume of scattering spheres and volume of the scattering spheres plus the volume of distilled water that is added to dilute the suspension. The number density of spheres in a unit volume of concentration η is $N_0=\eta/V_p$, where V_p is the volume of a single sphere, $\pi D^3/6$. Thus, the optical thickness of the purely scattering medium may be written as

$$\tau=(\eta/V_p)C_{sca}\ell=\eta c'\ell \tag{2}$$

where c', the effective scattering coefficient, equals C_{sca}/V_p and has units of cm^{-1}.

One determines c' experimentally by measuring the transmission through a cell containing distilled water and the scattering media relative to the transmission through another cell containing just distilled water. In the pure distilled water case, the power transmitted through the cell is closely approximated by the expression

$$I_w=I_0(1-\rho_1)(1-\rho_2)e^{-\tau_w} \tag{3}$$

where ρ_1 and ρ_2 are the front and back cell wall reflectances, respectively, τ_w the optical thickness of the distilled water, and I_0 the incident radiation. In the distilled water-scattering media case, the corresponding expression is

$$I_s=I_0(1-\rho_3)(1-\rho_4)e^{-\tau_w-\tau} \tag{4}$$

where ρ_3 and ρ_4 are the front and back cell wall reflectances of a second cell and are assumed to be different than ρ_1 and ρ_2 (i.e., the cells are not truly identical). τ is the optical thickness

Table 1 Comparison of experiment and theory

D, μm	x	T	η	T_0	c'(exp)	c'(the)
0.03	0.198	0.175	0.04237		36	
		0.330	0.02119	0.819	43	33
		0.520	0.10590		43	
		0.670	0.00530		38	
0.109	0.7197	0.702	0.0030		1140	
		0.780	0.0020	0.9883	1180	
		0.878	0.0010		1180	1435
		0.847	0.000821		1260	
		0.892	0.000410	0.939	1260	
0.312	2.060	0.455	0.000821		9430	
		0.670	0.000410	0.974	8940	12540
		0.805	0.000205		9270	
0.481	3.176	0.292	0.000600		20517	
		0.534	0.000300	1.000	20912	22982
		0.760	0.000133		20583	
0.500	3.301	0.190	0.000821		19320	
		0.375	0.000410	0.928	22071	24070
		0.590	0.000205		22070	
0.527	3.480	0.085	0.000821		29060	
		0.275	0.000410	0.924	29520	25588
		0.505	0.000205		29430	
		0.695	0.000102		27750	
0.801	5.289	0.170	0.000410		40350	
		0.378	0.000205	0.891	41770	36470
		0.578	0.000102		41180	
1.011	6.676	0.254	0.000400		34210	
		0.480	0.000200	0.998	36598	40247
		0.782	0.000067		36585	
2.02	13.338	0.428	0.000410		17613	
		0.690	0.000102	0.882	23920	22864
		0.780	0.000051		23940	
5.7	37.637	0.813	0.000410		3660	
		0.578	0.000205	0.945	3580	5412
		0.878	0.000102		3680	
25.7	169.69	0.912	0.000410		827	
		0.927	0.000205	0.943	830	1245
		0.935	0.000102		830	

resulting from the addition of the scattering media. It is also assumed here that the effects of the water and the scattering media are directly separable. The ratio of these powers is the transmittance. That is,

$$T=\frac{I_w}{I_s}=\frac{(1-\rho_1)(1-\rho_2)e^{-\tau}}{(1-\rho_3)(1-\rho_4)}=T_0e^{-\tau} \tag{5}$$

Most of the time an expensive effort is made to purchase cells that are "identical,"—high-grade optical quality cells with antireflection coatings. Unfortunately, this is not always possible. Thus, the factor T_0 must be handled carefully since, in general, its magnitude is not one. So, by combining Eqs. (2) and (5) one can determine c' from experiment as

$$c'(\text{exp})=1/(\eta\ell)\ln(T_0/T) \tag{6}$$

For the latex particles used, one can write the theoretical value of c' as

$$c'(\text{the})=C_{\text{sca}}/V_p=3Q_{\text{sca}}/(2D)=(3\pi/2\lambda)Q_{\text{sca}}/X \tag{7}$$

where C_{sca} and Q_{sca} are quantities which may be determined using Mie theory.

The index of refraction of water and polystyrene latex depends on wavelength. The expression which may be used for water[4] is

$$n_w=[1.7521+0.00811/\lambda_0^2]^{1/2} \tag{8}$$

and for polystyrene latex[5] is

$$n_L=1.5683+0.010087/\lambda_0^2 \tag{9}$$

Table 2 Cell-pair effects on T_0

Cell pair	D, μm	T_0
1	1.011	0.998
	0.481	1.00
	0.109	0.988 (1st part)
2	0.312	0.974
3	5.7	0.945
	25.7	0.943
4	0.109	0.939 (2nd part)
5	0.500	0.928
	0.527	0.924
6	2.02	0.882
	0.801	0.891
7	0.03	0.819

where λ_0 is the vacuum wavelength in microns. Since all of the data being presented here have been taken at one wavelength, 0.6328 μm, only $n_w=1.3313$ and $n_L=1.5935$ (a relative index of 1.1969) were used.

Latimer[6] examined the effects of finite sized photometer optical systems on transmission measurements. The resultant correction factor is a function of the system acceptance angle, the particle size parameter, and the relative index of refraction. This approximate correction factor was not used in this study to reduce transmission data because it was assumed that the acceptance angle of the detection system used was small enough such that this correction would be negligible.

Results

The comparison of experimental and theoretical values of c' as a function of scattering sphere size are given in Table 1

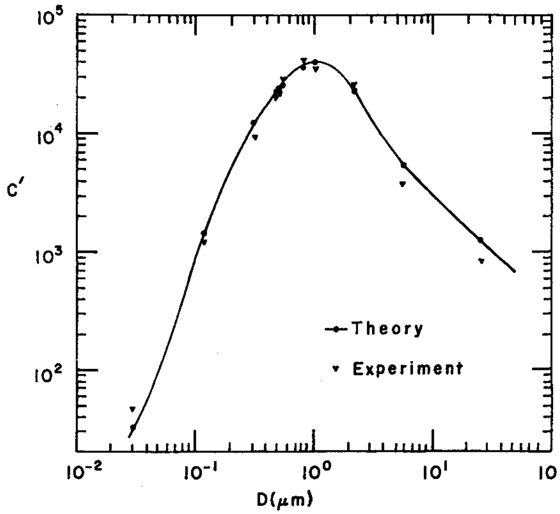

Fig. 1 Comparison of experimental and theoretical effective scattering coefficients as a function of particle size for a wavelength of 0.6328 μm.

for 11 different sphere diameters ranging from 0.03 to 25.7 μm and 7 different cell-pairs. These data were accumulated over a 2-yr period by three different experimenters using the same procedure. T, η, and ℓ were measured, D was given by the manufacturer, and c' and T_0 were calculated. Perusal of this table indicates fair agreement of the experimentally determined c'(exp) with its theoretical counterpart, c'(the).[7] Do not be confused by the apparent indication that T_0 is a function of particle diameter. This is not the case. Table 2 presents the pertinent information that describes the variation of the factor T_0, and the 7 cell-pairs used. Notice that cell-pair 1, which were in fact high quality cells, produced T_0 values of approximately one (the wall reflectances were essentially zero); all of the others did not (they were not high quality cells).

Figure 1 illustrates the results pictorially. From the figure it can be seen that the effective scattering coefficient c' behaves in the proper fashion both in terms of shape and magnitude. In fact, from Eq. (7) and using an asymptotic formula from Ref. 8,

$$c'(\text{the}) \propto Q_{\text{sca}}/x \rightarrow x^3 \quad \text{for } x \ll 1$$

and recalling that Q_{sca} approaches a constant for large x,

$$c'(\text{the}) \propto Q_{\text{sca}}/x \rightarrow 1/x \quad \text{for } x \gg 1$$

Even though the percentage error is large for the very small and the very large-sized particles, the agreement is good considering the wide range in the magnitude of the quantities involved (i.e., $0.03 \leq D \leq 25.7$ or $0.19 \leq x \leq 170$, and $33 \leq c' \leq 40{,}250$). This error is more than likely the result of two effects. First, the spheres are a size slightly different than indicated by the manufacturers. This has been observed before.[9] Second, the spheres are not exactly uniform in size. Some slight distribution exists which would have an effect on the value of c'(the), particularly for larger particles.

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References

[1] Look, D. C., Nelson, H. F., Crosbie, A. L., and Dougherty, R. L., "Two-Dimensional Multiple Scattering: Comparison of Theory and Experiment," *Journal of Heat Transfer,* Vol. 100, Aug. 1978, pp. 480-485.

[2] Look, D. C., "Two-Dimensional Multiple Scattering from a Medium of Finite Thickness," *Journal of Heat Transfer,* Vol. 101, Aug. 1979, pp. 556-557.

[3] Look, D. C. and Lackner, M. F., "Two-Dimensional Scattering in an Absorbing Medium: Preliminary Experiments," *Letters in Heat and Mass Transfer,* Vol. 6, Sept./Oct. 1979, pp. 385-395.

[4] Maron, S. H., Pierce, P. E., and Ulevitch, I. N., "Determination of Latex Particle Size by Light Scattering-IV," *Journal of Colloid Science,* Vol. 18, 1963, pp. 470-482.

[5] Delzelic, G. and Kratohvil, J. P., "Determination of Particle Size of Polystyrene Latexes by Light Scattering," *Journal of Colloid Science,* Vol. 16, 1961, pp. 561-580.

[6] Latimer, P., "Dependence of Extinction Efficiency of Spherical Scatters on Photometer Geometry," *Journal of the Optical Society of America,* Vol. 62, No. 2, 1972, pp. 208-211.

[7] Nelson, H. F., private communication, Mech. Engr. Dept., UMR.

[8] Wickramasinghe, N. C., *Light Scattering Functions for Small Particles,* John Wiley & Sons, New York, 1973.

[9] Wallace, T. P. and Kratohvil, J. P., "Comments on the Comparison of Scattering of Coherent and Incoherent Light by Poly-Dispersed Spheres with Mie Theory," *Applied Optics,* Vol. 8, No. 4, 1969, pp. 824-826.